Economics 792. Static discrete choice labour supply

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Stochastic discrete choice labour supply models are very popular in empirical research and have frequently been used as a tool to perform tax policy analysis. In this problem set we will examine the basic estimation of this class of model, and for simplicity we will abstract from any demographic heterogeneity (although it is straightforward to incorporate this).

Suppose that individuals have the choice to work $h \in [0, 10, 20, 30, 40]$ hours per week. Preferences over these discrete alternatives may be described by a parametric utility function:

$$U(c, h) = \gamma \times \left[\frac{c^\theta}{\theta} - \alpha h \right] + \epsilon_h,$$

where the state specific errors ϵ_h are assumed to follow a Type-I extreme value distribution. Consumption is given by $c = y + wh - T(wh)$, where y is non-labour income, $T(\cdot)$ is the tax system, and w is the gross hourly wage rate generated by the following log-linear relationship:

$$\log w = \mu_w + \epsilon_w,$$

and where the unobserved component of wages ϵ_w is Normally distributed with mean 0 and standard deviation σ_w.

1. Suppose that the parameter values are: $\boldsymbol{\theta} = \{\sigma_w = 0.55, \mu_w = 1, \theta = 0.3, \alpha = 0.1, \gamma = 2\}$ and that $y \sim \text{Uniform}[10, 100]$. Suppose also that earnings below \$80 per week are not taxed; any earnings greater then \$80 are taxed at the constant marginal tax rate $\tau = 0.3$. Non-labour income is not taxed. Under these assumptions, simulate a dataset of 1,000 observations. What is the distribution of work hours in your simulated dataset? How does the distribution of non-labour income and (offered) wages vary with work hours?

2. It is well-known that a simple OLS regression of wages performed on a sample of workers will typically be inconsistent. In the context of the model presented here, briefly discuss how the size of the sample selection bias will vary with the parameter γ.

3. Write down the log-likelihood function for this model, remembering that wages will be unobserved for non-workers.

4. Write your own computer code (in a language of your choice) to estimate the model using the dataset you generated above (which comprises, work hours, non-labour income, and wages only for workers). Having estimated the model, how close are your parameter estimates $\hat{\boldsymbol{\theta}}$ to the true values that you used to generate the dataset?

5. Using the true parameter values (i.e. those given in part 1), what is the impact on the distribution of work hours of changing the tax system so that all earnings are taxed at the constant marginal tax rate 0.2? How does the amount of tax revenue raised by the government change? Can you determine the value of the constant marginal tax rate which provides the same amount of revenue as the progressive schedule described in part 1?