Economics 792. Family labour market search

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Consider a unitary household comprised of two adults, where each household member sequentially samples job offers. The arrival rate of job offers for adult $i \in \{m, f\}$ is λ_i^e when employed and λ_i^u when unemployed. Regardless of their labour market state, individual i samples wage offers from the distribution $F_i(\cdot)$. When employed, adult i loses his/her job at exogenous rate δ_i. The continuous time discount rate is ρ. The household has non-labour income $z > 0$. An unemployed individual receives no other income. The flow utility function is given by:

$$v(y, P_m, P_f) = \frac{y^{1-\gamma}}{1-\gamma} + \alpha_m(1 - P_m) + \alpha_f(1 - P_f),$$

where $P_i = 1$ if i is employed, zero otherwise, and $y = z + w_m P_m + w_f P_f$ is total flow income.

1. Write down the value functions that characterise the optimal behaviour of the couple. These should be presented in a similar form to what we considered in the lectures, but here appropriately modified by the presence of on-the-job search and gender asymmetry.

2. Assume that $\lambda_m^u = 0.1$, $\lambda_m^e = 0.05$, $\lambda_f^u = 0.2$, $\lambda_f^e = 0.15$, $\delta_m = 0.005$, $\delta_f = 0.005$, $\rho = 0.001$, $z = 0.1$, $\gamma = 0.5$, $\alpha_m = 0.8$, $\alpha_f = 1$. The distributions of wage offers F_i is truncated log-normal with $\text{Supp}(F_m) = [0.1, 5]$ and $\text{Supp}(F_f) = [0.1, 7]$, and with respective location and scale parameters $\mu_m = -0.1$, $\sigma_m = 0.5$, $\mu_f = 0$, $\sigma_f = 0.65$. Numerically solve the value functions that you characterised in (1) using a programming language of your choice.

3. Plot the value functions you calculated (as a function of the male/female wage) in the joint *ue* and *eu* states. On your figure also show the value in the joint *uu* state. What is the reservation wage for each adult i when their spouse is unemployed?

4. Now suppose that that the husband is employed at some wage w_m and that his unemployed wife receives her highest possible wage offer $w_f = \overline{w}_f$. Are there any values of w_m such that the wife would accept her offer and her husband would simultaneously choose to quit his job?

5. Using your computed value functions, simulate labour market histories for 5000 couples and report the implied (approximate) steady state fraction of couples in the joint *uu*, *ue*, *eu* and *ee* states.

6. Write the flow equations that characterise the steady state of the economy. Hint: your expressions should characterise the joint states m_{uu}, $m_{ee}g_{ee}(w_m, w_f)$, $m_{eu}g_{eu}(w_m)$ and $m_{ue}g_{ue}(w_f)$, where m_{jk} is the fraction of couples in joint state jk, and g_{jk} is the earnings density.

7. **Optional.** Numerically solve the flow equations from (6). How close are the values of m_{uu}, m_{eu}, m_{ue} and m_{ee} to those that you obtained from the simulation in (5)?